**FEM Problem Set B**

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1. We want to solve the Poisson equation:
2. A variational formulation for the Poisson equation is derived here.

Where the Hilbert space is defined to be

With the following associated norm , the bilinear form and the linear form .

1. We want to prove the Lax-Milgram theorem on this variational formulation.

Because of the Lax-Milgram theorem, we know that there exists a unique solution to the variational problem.

1. We formulate the following abstract Galerkin method.
2. We want to prove that .
3. We want to solve the following problem with and .
4. We formulate the variational formulation:

Thus, we formulate the Galerkin FEM as following.

Mesh: The subdivision of the interval by the points , , .

Space:

Weak form: Find such that .

1. Let be the solution to the following dual problem of this .
2. We have the following equation.

From Poincare’-Friedrichs’ inequality, we know that:

We will then have the following procedure: